Exercises 3

Exercise 3.1

A particle is confined in a linear box of length L surrounded by walls of infinite potential. The ground state of this system is described by the following wave function:

$$\Psi_1(x) = \sqrt{\frac{2}{L}} \times \sin\left(\frac{\pi x}{L}\right)$$

a) What is the probability of finding the particle at a given position x?
b) At which position is the maximum probability density?
c) What is the total probability of finding the particle in the box?
d) If L = 10 nm, what is the probability that the particle is between 4:95 and 5:05 nm?

Note: Exercise 3.1 will be solved on the board during the exercise session this Friday, September 27, 2024.

Exercise 3.2

The total energy of the particle in the box can be calculated as

$$E_{\text{tot}} = E_{\text{kin}} + E_{\text{pot}},$$

where the kinetic energy is given by

$$E_{\text{kin}} = \frac{1}{2}mv^2.$$

Write down an expression for the total energy of the particle in the box, using the de Broglie relationship ($p = mv = \frac{h}{\lambda}$) and the fact that the wavelength must satisfy $\lambda = \frac{2L}{n}$.
What is the main implication of this equation?

Exercise 3.3

True or False?

a) The ground state energy of a particle in a box (PDB) is zero.
b) The energy levels of the PDB are equidistant.
c) Increasing the steady-state energy of the PDB is equivalent to increasing the number of nodes in the wave function.
d) All solutions of the time-independent Schrödinger equation for the PDB are allowed steady-state wave functions.
e) The transition of the PDB that absorbs the longest wavelength photon is from the n = 1 level to the n = 2 level.

Exercise 3.4

The concept of quantization of energy is foundational in quantum mechanics. In atomic systems, electrons can only occupy specific, quantized energy levels. However, when a photon with energy greater than the difference between two energy levels interacts with an atom, the electron can transition to a higher energy level, and the excess energy becomes kinetic energy of the electron.

Given: The energy levels of the hydrogen atom are described by the formula:

$$E_n = -\frac{13.6\ eV}{n^2}$$

a) Calculate the energy of the first two energy levels (n=1 and n=2) of the hydrogen atom.
b) If the electron in the hydrogen atom absorbs a photon with an energy of 12 eV while in the ground state (n=1), to which energy level, if any, will the electron transition? Calculate the kinetic energy acquired by the electron due to the excess energy from the photon.
c) Based on your results, discuss the implications for atomic systems when they interact with high-energy photons.